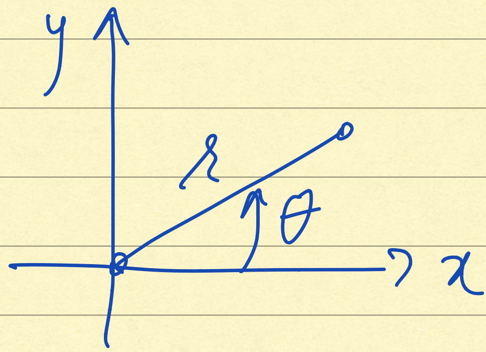
Mudança de Variáveis:

1) polares (r,θ)



$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \arctan\frac{y}{x} \end{cases}$$

r

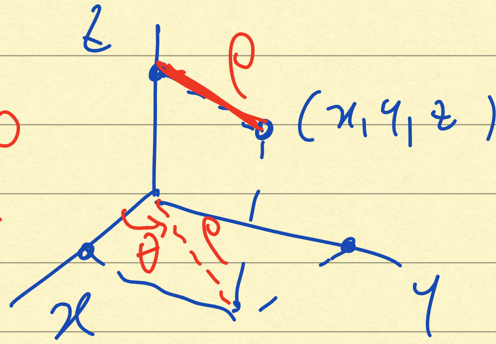
$$\iint_X dx\,dy \equiv \iint_T r\,dr\,d\theta$$

2) cilíndricas (ρ,θ,z)

$\sqrt{x^2+y^2} = \rho$

|||

distância ao eixo Oz.

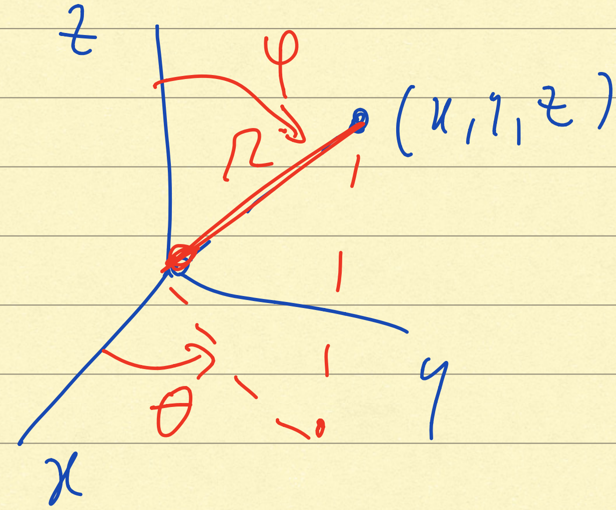


$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \\ z = z \end{cases}$$

ρ

$$\iiint_X dx\,dy\,dz = \iiint_T \rho\, d\rho\, d\theta\, dz$$

3) <u>Esféricas</u>: (r, θ, φ)



$r = \sqrt{x^2+y^2+z^2}$

$|||$

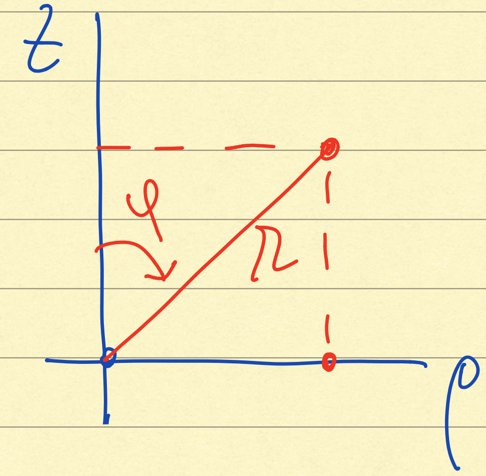
distância à origem.

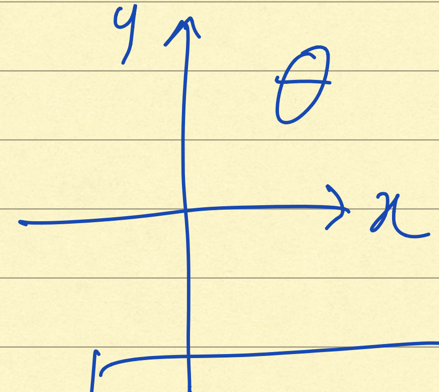
$$\begin{cases} x = r \operatorname{sen}\varphi \cos\theta \\ y = r \operatorname{sen}\varphi \operatorname{sen}\theta \\ z = r\cos\varphi \end{cases}$$

$$\boxed{r^2 \operatorname{sen}\varphi}$$

$$\iiint_X dx\,dy\,dz = \iiint_T r^2 \operatorname{sen}\varphi\, dr\, d\theta\, d\varphi$$

Cilíndricas e esféricas:





$$\boxed{\theta = \arctan \frac{y}{x}}$$

$$\rho = \sqrt{x^2+y^2} \qquad r = \sqrt{x^2+y^2+z^2}$$

$$\rho = r \operatorname{sen} \varphi \qquad r = \sqrt{\rho^2+z^2}$$

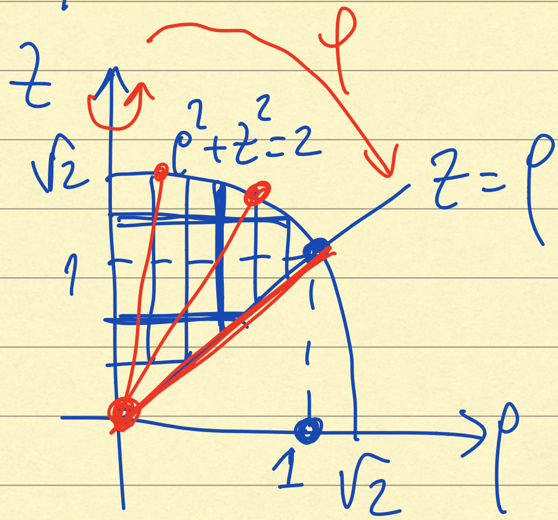
$$z = r \cos \varphi$$

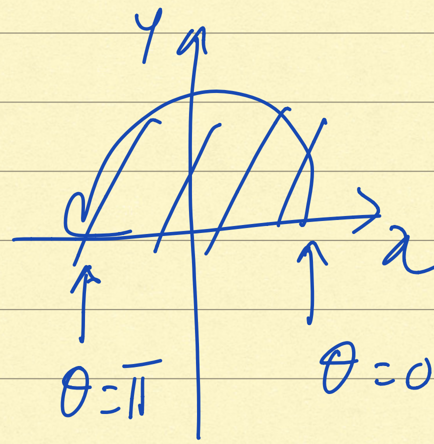
Exemplo: X:
$$x^2+y^2+z^2 \leq 2$$
$$z \geq \sqrt{x^2+y^2}$$
$$y \geq 0$$

$$\underbrace{x^2+y^2}_{\rho^2}+z^2 \le 2 \longrightarrow \rho^2+z^2 \le 2$$

$$z > \sqrt{x^2+y^2} \longrightarrow z > \rho$$

$$y > 0 \longrightarrow \boxed{0 < \theta < \pi} \checkmark$$



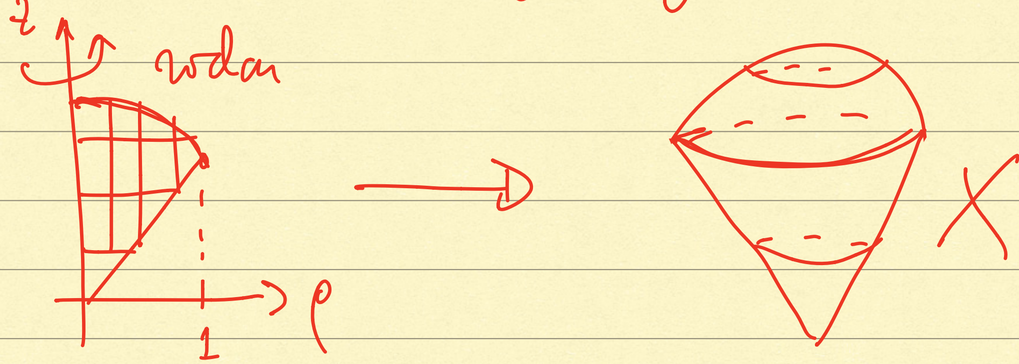


$$\mathrm{Vol}_3(X) = \int_0^{\pi} \left(\int_0^{1} \left(\int_{\rho}^{\sqrt{2-\rho^2}} \rho \, dz \right) d\rho \right) d\theta$$

$$= \int_0^{\pi} \left(\int_0^{1} \rho \left(\sqrt{2-\rho^2} - \rho \right) d\rho \right) d\theta$$

...

$$\mathrm{vol}_3(X) = \int_0^{\pi} \left(\int_0^{\pi/4} \left(\int_0^{\sqrt{2}} r^2 \operatorname{sen}\varphi \, dr \right) d\varphi \right) d\theta$$

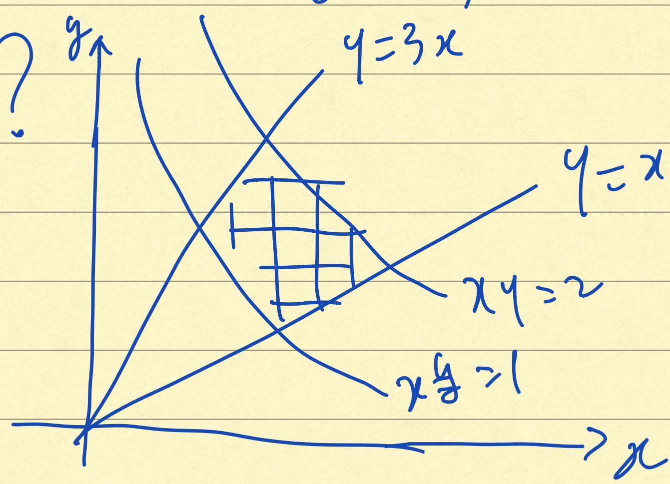


$\boxed{\text{Outras Coordenadas}}$

<u>Exemplo</u>:

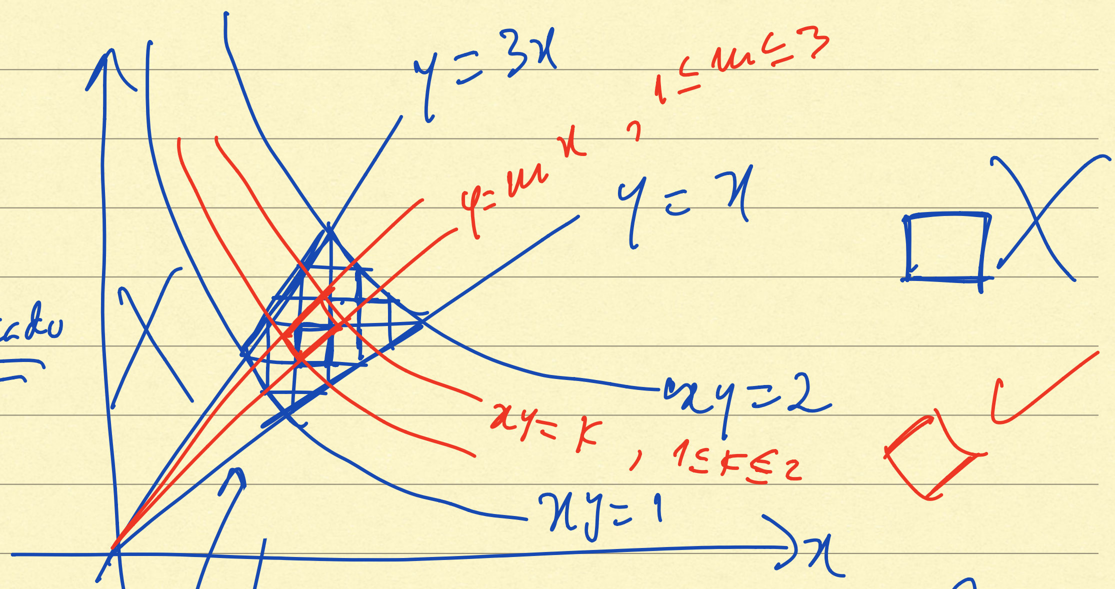
$$X = \{ (x,y) \in \mathbb{R}^2 : \; 1 \leq xy \leq 2 \; ; \; x \leq y \leq 3x \}$$

$$\mathrm{vol}_2(X) = \; ?$$



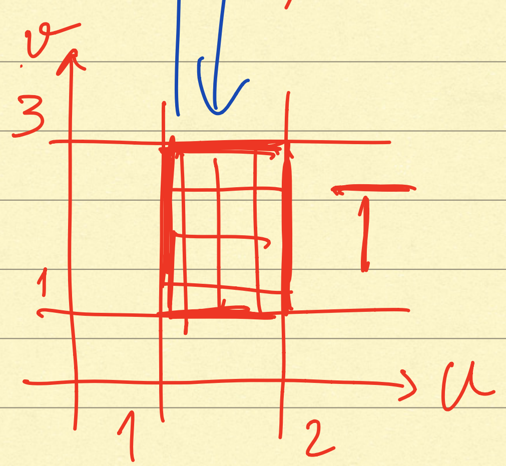
<u>Complicado</u>

Complicado



$$\boxed{u = xy \quad g \quad g^{-1} \quad v = \frac{y}{x}} \longrightarrow g \,? \text{ ou } g^{-1}\,?$$

$1 \leq u \leq 2 \quad ; \quad 1 \leq v \leq 3$



(simples)

$$x = g(t)$$

velhas   novas

$t = (u, v)$ $\qquad\qquad$ $x = (x, y)$

novas $\qquad\qquad$ velhas

$$x = g(t)$$

$$\begin{cases} u = xy \\ v = \dfrac{y}{x} \end{cases} \qquad \boxed{t = g^{-1}(x)} \xrightarrow{x = g(t)}$$

$$\int_X f(x)\,dx = \int_T f(g(t))\underbrace{|\det Dg(t)|}_{???}dt$$

Relação entre $\boxed{\det Dg}$ e $\det Dg^{-1}$?

$$t \longmapsto x = g(t)$$

$$g^{-1}(x) = t \longleftarrow\!\shortmid\; x$$

$$x = g(t) \qquad t = g^{-1}(x)$$

$$x = g\left(g^{-1}(x)\right)$$

$$I = Dg\left(g^{-1}(x)\right) Dg^{-1}(x)$$

$$1 = \boxed{\det Dg\left(g^{-1}(x)\right)} \det Dg^{-1}(x)$$

$$\boxed{\det Dg(t) = \frac{1}{\det Dg^{-1}(x)}}$$

$$\begin{cases} u = xy \\ v = \dfrac{y}{x} \end{cases} \qquad \begin{aligned} 1 &\le xy \le 2 \\ 1 &\le \frac{y}{x} \le 3 \end{aligned}$$

$$g^{-1}(x,y) = \left(xy, \frac{y}{x}\right)$$

$$\det D g^{-1}(x,y) = \det \begin{bmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{bmatrix} = \frac{y}{x} + \frac{y}{x}$$

$$= 2\left(\frac{y}{x}\right)$$

$$= 2v$$

$$\mathrm{vol}_2(X) = \iint_X dx\,dy$$

$$= \iint_T \frac{1}{2v}\,du\,dv$$

$$\mathrm{vol}_2(X) = \int_1^2 \left(\int_1^3 \frac{1}{2v}\,dv \right) du$$

$$= \frac{1}{2}\int_1^2 \ln 3 \, du = \frac{\ln 3}{2} \quad //.$$

—————— // ——————

$$\int_X f \quad \begin{cases} \to \underline{\text{Fubini}} \\ \to \underline{\text{Mud. de variável}} \end{cases}$$

$X \subset \mathbb{R}^4$?

$f : \mathbb{R}^4 \to \mathbb{R}$?